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SUPERPOSITION OF NEUTRINO EIGENSTATES AND NEUTRINO OSCILLATION

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The possibility exists that the weak interaction between neutrino and antineutrino produces states which are superpositions of left-handed neutrino and right-handed antineutrino. This new states do not have well-defined lepton number. The peculiar properties of neutrinos and their oscillation in this case most probable are indicative of the Majorana nature of neutrinos.

The investigation has been performed at the Laboratory of Particle Physics, JINR

Суперпозиция собственных состояний нейтрино и осцилляции нейтрино

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Показано, что в результате слабого взаимодействия между нейтрино и антинейтрино возможно существование таких состояний, которые являются суперпозицией левого нейтрино и правого антинейтрино. Эти новые состояния не имеют определенного лептонного числа. Свойства этих нейтрино и их осцилляции наиболее вероятно свидетельствуют о майорановской природе нейтрино.

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One of the fundamental questions in modern physics is: under what conditions can neutrino oscillation occur in the vacuum?

The phenomenon of neutrino oscillation arises from a mismatch between the weak eigenstates and the eigenstates of neutrino mass [1]. In the case of possibly massive neutrinos the relative phase change between the mass eigenstates with time can show an oscillation pattern [2]. This oscillation is extremely sensitive to the mass of the neutrino and the mixing angle.

In this paper the study of neutrino oscillation is made taking into consideration the superposition of neutrino and antineutrino eigenstates. Analogously to *K*-meson mixing, this oscillation is not sensitive to the mass of neutrino but to the mass difference of the new states produced and not having a well-defined lepton number.

Let's consider a two-state quantum-mechanical system having identical quantum numbers, characterized by a Hamiltonian H. The initial states S_1 and S_2 will develop in time following the evolution equation:

$$ih\frac{\partial S_1}{\partial t} = H_1 S_1 \quad \text{and} \quad ih\frac{\partial S_2}{\partial t} = H_2 S_2.$$

The solutions of these equations are:

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$$S_1 = k_1 \exp(-iH_1 t/h), \qquad S_2 = k_2 \exp(-iH_2 t/h).$$

Each of those states would phase-rotate in its own eigenstate. Let's now assume an interaction term in the Hamiltonian in the form:

$$ih\frac{\partial S_1}{\partial t} = HS_1 + aS_2 \text{ and } ih\frac{\partial S_2}{\partial t} = HS_2 + aS_1.$$

The amplitudes of states S_1 and S_2 starting from a state $S_1(t=0)$ are given by: $S_1(t) = \exp(-iHt/h)\cos(at/h)$; $S_2(t) = \exp(-iHt/h)\sin(at/h)$. The new states $(S_1 + S_2)$ and $(S_1 - S_2)$ will have different eigenvalues $(H+a,\ H-a)$. This system will permanently move from state S_1 to S_2 and back.

Let's now consider two states having different quantum numbers. If an interaction exists between these states, the time evolution of the system will be described by a linear superposition of states and will not have a well-defined relevant quantum number. A more familiar example is that of K^0 and $\overline{K}^0$-oscillation [4]. Under the action of weak interaction the original strong interaction eigenstates of K^0 and $\overline{K}^0$-mesons are mixed. The weak eigenstates in this case are the linear superpositions of K^0, $\overline{K}^0(K_1, K_2)$. If both K_1 and K_2 mesons were stable particles, then K_1 and K_2 would each evolve with respect to its own Hamiltonian and after time t the relative phase K_1 and K_2 (present in the original K^0, $\overline{K}^0$-states) would have changed. The new states would thus be the new linear combination of K^0, $\overline{K}^0$ and these states would also not have well-defined strangeness. The relative phase change of K_1, K_2 with time will be the result of this oscillation. In reality the weak interaction also causes the decay of K_1, K_2. Due to the action of the weak interaction K_1, K_2 mix and CP-violation occurs. Owing to the decays of K_1, K_2 the relative amplitude of K_1, K_2 present in the original K^0, $\overline{K}^0$-mesons would change with time. So not only the relative phase but also amplitude of K_1, K_2 in K^0, $\overline{K}^0$ mesons change with time. Under the weak fermion interaction a pair of s-quarks converts into a pair of d-quarks and back. The CP-non-invariant interaction gives rise to K_1, K_2 mixing, while the CP-invariant part of this interaction contributes to the mass difference.

An important aspect of the phenomenon is that the interaction connecting two neutral states with different quantum numbers violates the conservation laws involved and, due to that the superposition transition can appear. The mixing may not take place if interaction is forbidden. For instance, conservation of electrical charge forbids the oscillation between a proton and an antiproton.

Thus, the superpositional states will appear if the base states are neutral and are of equal masses and an interaction between this states violates those quantum numbers that aren't their eigenvalue. This is not the case for the neutrino. Electron-type of neutrino cannot be coupled to a muon-type of neutrino, unless an interaction exists, which violates lepton numbers. This interaction would play for neutrinos the same role as weak interaction plays for kaons. But we know that lepton numbers conserve to a very high degree. Now one needs to clarify which kind of coupled states could be between two neutrino states taking into account the fact that neutrinos are fermions. The weak eigenstates of neutrinos do not coincide with the mass eigenstates, and due to that the components of weak eigenstates can be expressed in terms of the mass eigenstates as:

$$\nu_i = \sum u_{ij}^* \nu_j \,. \qquad (1)$$

Eigenstates of neutrinos in the limit in which the neutrino mass is degenerate would be coherent and could therefore interfere. If the masses of the different eigenstates were too far apart, then the velocities would so differ that the states could lose the interference picture. The time evolution of the neutrino can be described by the coherent sum:

$$\nu_i(t) = \sum_{j=1}^{N} \exp\left(\left(-i\sqrt{p^2 + m_\nu^2}\right) t\right) u_{ij}^* \nu_j \,. \qquad (2)$$

One does not detect the individual terms in (2) but their linear combination corresponding to certain weak eigenstates. Only eigenstates of ν and $\bar{\nu}$ preserve the interference pattern. The way to detect neutrinos and possible neutrino oscillations is through its weak interaction. So, two neutral degenerate levels of neutrino and antineutrino of definite flavour under the action of weak interaction can be described by a linear superposition of states neutrino and antineutrino. This system will continuously move from the left-handed state to the right-handed and vice versa. So, a neutrino of definite flavour, produced at $t = 0$, with respect of (1) can be expressed by a linear superposition of the left-handed neutrino and right handed antineutrino. These new states will not have a well-defined chirality, since they are linear combinations of left and right chiral projections. However, they can be eigenstates of helicity ($\sigma * \mathbf{p}/p$). Obviously, for zero mass, helicity and chirality eigenstates coincide. If the mass is non zero the chirality will differ from the helicity by the term m_ν^2/E^2. In the case of Dirac type of neutrinos the opposite chirality transitions between neutrino and antineutrino are suppressed by a factor m_ν^2/E^2. The suppression is related to the helicity and fermion number conservation [5]]. For the Majorana type of neutrinos there is no helicity violation because the Majorana condition requires a neutrino to consist of a left-handed 2-spinor together with a charge conjugate right-handed one. The mixing of two degenerate levels in the vacuum must result in level splitting. These new levels would have definite CP-parity.

$$\nu_1 = \frac{\nu + \bar{\nu}}{\sqrt{2}} \quad \text{and} \quad \nu_2 = \frac{\nu - \bar{\nu}}{\sqrt{2}} \,.$$

Those states after a time t will have:

$$\frac{1}{\sqrt{2}}\left[\nu_1 \exp\left(-im_1 t - \Gamma_1 t/2\right) + \nu_2 \exp\left(-im_2 t - \Gamma_2 t/2\right)\right].$$

If both states were stable ($\Gamma_1 = \Gamma_2 = 0$), then after $t = \pi/\delta m_{12}$ the beam of neutrino would consist of antineutrinos. After another time interval the beam of antineutrinos would change chirality and would consist of neutrinos. The new states thus reached could now be a new linear superposition of neutrino and antineutrino. Experiments of this type make it possible to determine the mass difference δm_{12}. Two states ν_1 and ν_2 are eigenstates of CP-parity. If they were both unstable, then under the action of the weak interaction they could be mixed with and violate the CP-parity. Such oscillations would be suppressed by exponential damping due to $\Gamma_{12} \neq 0$. The probability will be:

$$P_{\nu_1 \rightleftarrows \nu_2}(t) = \frac{1}{4}\left\{ \exp(-\Gamma_1 t) + \exp(-\Gamma_2 t) \pm 2\exp\left[-(\Gamma_1 + \Gamma_2)\, t/2\right] \cos \delta m_{12} t \right\}.$$

The mass difference δm_{12} does not depend on the mode of decay. Because of small phase space available for the ν_1, ν_2-decays, one expects the width, $(\Gamma_1 + \Gamma_2) \geq \delta m_{12}$, to enhance transition.

So, the transformation of left-handed neutrinos to right-handed antineutrinos and vice versa forms the objects having only two independent components. This transition violates lepton number by two units and conserves CP-parity. The Majorana nature of neutrino oscillation is most probable. Recently there was observed semileptonic decay of tau-lepton claimed for the Majorana type of neutrino [6].

From the foregoing consideration, it is seen that two Majorana neutrinos ν_1 and ν_2 probably would have a tiny mass difference as a result of $\Delta L = 2$ transition. For the detection of ν-oscillation one considers the so-called «appearance experiments» to look for the presence of antineutrinos in ν-beam or vice versa. It is obvious that the ability to detect ν-oscillations depends on the mass difference of Majorana neutrinos ν_1 and ν_2 and they would be stable or unstable particles. If neutrinos ν_1, ν_2 were stable, then after time $\pi / \Delta m_{1,2}$ two fluxes will appear: of neutrino and of antineutrino. The aim is to measure the flux of neutrino of opposite sign at different distances from the source. The mass difference can be measured directly in this case. To perform an experiment the detector has to be sensitive to both signs of neutrino. For example: if source is the source of antineutrino, then one needs to detect the flux of antineutrino at given distances via inverse beta reaction:

$$\tilde{\nu} + p \to n + e^+$$

and elastic scattering

$$\tilde{\nu} + e \to \tilde{\nu} + e.$$

If no oscillation is present, then the ratio $N\tilde{\nu}p / N\tilde{\nu}e$ will be constant for any distances. If oscillation is present, then the ratio $N\tilde{\nu}p / (N\tilde{\nu}e + N\nu e)$ would be function of $\Delta m_{1,2}$. To verify this, ν-oscillation can be detected in laboratory from high-intensity beta source or accelerator. Another possibilities are in solar neutrino detection. Antineutrinos are not predicted by any solar model, thus observation of antineutrino would be of major significance. To explain deficit of solar neutrino flux, the mass difference has to be $< 10^{-17}$ MeV–10^{-19} MeV. Study of ν–$\tilde{\nu}$ oscillation can be also done with short- or long-baseline neutrino experiments. If ν_1, ν_2 are unstable particles, the yield of neutrinos of opposite sign would depend upon branching ratio of decay mode to different types of neutrinos and $\Gamma / \Delta m_{12}$ ratio.

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